Exam Kwantumfysica 2

Date	21 June 2012
Room	Tentamenhal 01 Blauwborgje 4
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the book or slides, nor other notes or books
- The weights of the exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

1a)	5	2a)	10	3a)	5
1b)	5	2b)	10	3b)	5
1c)	10	2c)	10	3c)	5
1d)	5	2d)	5	3d)	5
		2e)	10		

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

Exercise 1

(a) Explain what is the purpose or use of the addition of angular momentum and give the definition of a "good" quantum number.

(b) Use the table below to write down the Clebsch-Gordan decomposition of the state $|j_1, j_2; j, m\rangle = |2, 1; 1, 0\rangle$ in terms of the product states $|j_1, j_2; m_1, m_2\rangle$ and verify the convention $\langle j_1, j_2; j_1, j - j_1 | j, j\rangle > 0$.

Table 1: Clebsch-Gordan coefficients $\langle j_1, 1; m_1, m_2 | jm\rangle$

j	$m_2 = 1$	$m_2 = 0$	$m_2 = -1$
$j_1 + 1$	$\left[\frac{(j_1+m)(j_1+m+1)}{(2j_1+1)(2j_1+2)}\right]^{\frac{1}{2}}$	$\left[\frac{(j_1-m+1)(j_1+m+1)}{(2j_1+1)(j_1+1)}\right]^{\frac{1}{2}}$	$\left[\frac{(j_1-m)(j_1-m+1)}{(2j_1+1)(2j_1+2)}\right]^{\frac{1}{2}}$
j_1	$-\left[\frac{(j_1+m)(j_1-m+1)}{2j_1(j_1+1)}\right]^{\frac{1}{2}}$	$\frac{m}{[j_1(j_1+1)]^{\frac{1}{2}}}$	$\left[\frac{(j_1-m)(j_1+m+1)}{2j_1(j_1+1)}\right]^{\frac{1}{2}}$
$j_1 - 1$	$\left[\frac{(j_1-m)(j_1-m+1)}{2j_1(2j_1+1)}\right]^{\frac{1}{2}}$	$-\left[\frac{(j_1-m)(j_1+m)}{j_1(2j_1+1)}\right]^{\frac{1}{2}}$	$\left[\frac{(j_1+m)(j_1+m+1)}{2j_1(2j_1+1)}\right]^{\frac{1}{2}}$

(c) In case of a constant, uniform external electric field one has to calculate the matrix elements $\langle nlm|z|n'l'm'\rangle$. Show that rotational invariance implies that for $\Delta m = m' - m \neq 0$ these matrix elements vanish. Show that the behavior under parity transformations demands that for even $l + l'$ these matrix elements vanish. Write down which matrix elements are nonzero for $n = 2, n' = 1$.

(d) Explain why j is not a good quantum number in case of a constant, uniform external magnetic field and discuss what is the consequence for the time dependence of a state that has a given j at $t = 0$?

Exercise 2

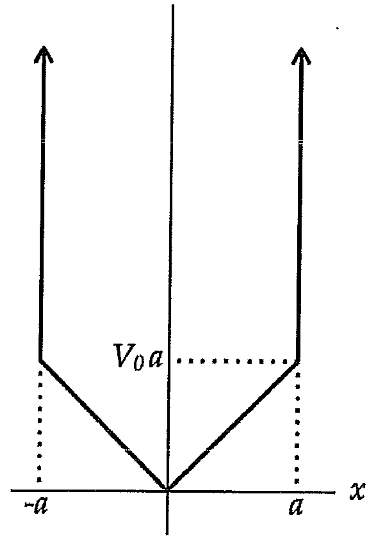
(a) Construct the solutions of the Schrödinger equation for the infinite square well potential,

$$V(x) = \begin{cases} 0 & -a \leq x \leq a \\ \infty & \text{elsewhere} \end{cases},$$

and give the allowed energies.

exercise 2 continued on the next page

Next consider the case $V(x) + V_0|x|$ for constant V_0 (see figure).



(b) Calculate in first order perturbation theory the correction to the ground state energy due to the addition of the potential $V_0|x|$ to $V(x)$. Give the condition(s) for which this perturbative result is valid.

(c) Obtain an upper bound to the ground state energy for the potential $V(x)+V_0|x|$ using the following normalized trial function,

$$\psi_T(x) = \sqrt{\frac{15}{16a^5}}(a^2 - x^2),$$

and show that it is larger than the perturbative result. Argue whether this is expected or not.

(d) Give an example of a trial function that would give an upper bound on the first excited state for $V(x) + V_0|x|$.

(e) Obtain the first two leading terms in $V_0 a/E$ for the allowed energies using the WKB method, assuming the ground state energy E_{gs} to be larger than $V_0 a$.
Recall that $(1 - x)^{3/2} = 1 - 3x/2 + 3x^2/8 + ...$

Exercise 3

Consider the Hamiltonian $H = H_0 + H'$, where the states $\psi_n^{(0)}$ form an orthonormal set of eigenstates of H_0 with energies $E_n^{(0)}$, i.e. $H_0\,\psi_n^{(0)} = E_n^{(0)}\,\psi_n^{(0)}$. H' is a perturbation acting from time $t_0 = 0$.

(a) Show that with the following expansion on the states $\psi_n^{(0)}$

$$\psi(t) = \sum_n c_n(t)\,\psi_n^{(0)}\,e^{-i\,E_n^{(0)}t/\hbar},$$

the coefficients satisfy

$$\dot{c}_m = \frac{1}{i\hbar}\sum_n H'_{mn}\,c_n(t)\,e^{i\,(E_m^{(0)}-E_n^{(0)})t/\hbar},$$

where $H'_{mn} = \langle\psi_m^{(0)}|H'|\psi_n^{(0)}\rangle$.

(b) Consider the particular case of a two-level system consisting of states a and b, with $H' = \theta(t)V$, where V is an $\vec{r}$-independent, t-independent potential. What is the probability that the system is in state b for $t > 0$ if for $t < 0$ it is in state a?

(c) Derive the same probability in first-order time-dependent perturbation theory for an $\vec{r}$-dependent, t-independent potential V.

(d) Show for the two cases in (b) and (c) whether the average energy is conserved or not.